NUMERICAL INVESTIGATION OF INDUCTION STAGE OF DEVELOPMENT OF NATURAL CONVECTION

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The process of development of convection in a two-dimensional region of square cross section, which is heated from below, is investigated numerically. The results are compared with earlier experimental results.

The process of development of natural convection in a two-dimensional fluid layer was investigated in [1] based on the measurements of nonstationary fields. It was shown that one of the characteristic features of this process is the existence of a period τ_* of the induction of convection. The experimental results obtained in [1] were generalized in the form of the following formula:

$$\tau_* = 70\,\mathrm{Ra}^{-2/3}\,\mathrm{Pr}^{-1/6} \quad \text{for} \quad \mathrm{Pr} \geq 10 \text{ and } \mathrm{Ra} > 10^4.$$

In the present article the process of development of convection is investigated numerically and the results are compared with the experimental results.

We investigated a region of square cross section filled with an inert fluid and included between two horizontal plates (y = 0 and y = h), held at constant temperatures T_1 and T_0 respectively with $T_1 > T_0$. The vertical plates (x = 0, x = h) are thermally insulated. At the initial time the temperature of the fluid is T_0.

The nondimensional equations describing the process have the following form:

$$\frac{\partial \vec{v}}{\partial \tau} + \mathrm{Pr}\,(\vec{v}\vec{\nabla})\vec{v} = -\vec{\nabla} p + \mathrm{Pr}\,\Delta\vec{v} + \mathrm{Ra}\,\theta\vec{i};$$

$$\frac{\partial \theta}{\partial \tau} + \mathrm{Pr}\,\vec{v}\vec{\nabla}\theta = \Delta\theta; \qquad \vec{\nabla}\vec{v} = 0. \tag{2}$$

The initial conditions are:

$$\theta(\eta,\ \xi,\ 0) = \vec{v}(\eta,\ \xi,\ 0) = 0; \tag{3a}$$

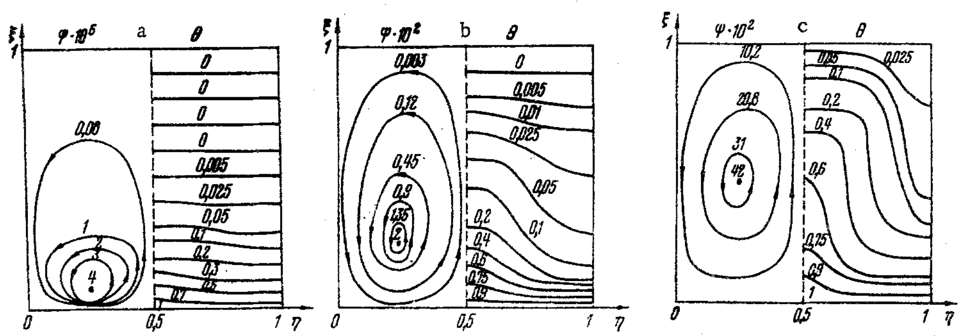

Fig. 1. Isotherms and stream functions (Ra = $1.7\cdot10^4$, Pr = 20): a) τ = $1.2\cdot10^{-2}$; b) $2.9\cdot10^{-2}$; c) $1\cdot10^{-4}$.

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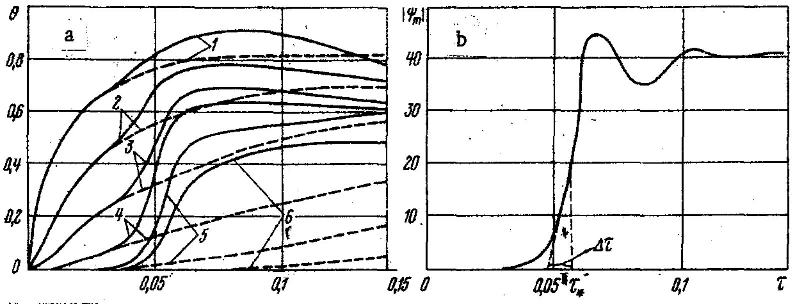

Fig. 2. Dependence of θ on τ (a) for different ξ: 1) $\xi = 0.1$; 2) 0.2; 3) 0.3; 4) 0.5; 5) 0.7; 6) 0.8 and dependence of $|\psi_m|$ on τ (b).

and the boundary conditions are:

$$\xi = 0,\quad \theta = 1,\quad \vec{v} = 0,\quad \xi = 1,\quad \theta = 0,\quad \vec{v} = 0,\quad \eta = 0;\ 1,\ \frac{\partial\theta}{\partial\eta} = 0,\ \vec{v} = 0, \tag{3b}$$

the coordinate $\xi = y/h$ is directed against the gravitational force.

The following quantities are chosen as the scales of distance, time, velocity, pressure, and temperature:

$$h,\ \frac{a}{h^2},\quad \nu/h,\quad \frac{\rho\nu a}{h^2},\quad (T_1 - T_0).$$

The system of equations (2)-(3a, b) was solved by the method discussed in [2]. Rayleigh's number was varied in the range 10^3-$5\cdot10^4$. Prandtl's number was kept constant in the computations and was equal to 20.

The pattern of development of convection is shown in Fig. 1 for Ra = $1.7\cdot10^4$. Since the pattern is symmetric, isolines of the stream function are shown on the left and isotherms on the right. In the beginning at the bottom surface of the fluid the convection motion appears in the form of two vortices rotating in opposite directions. The values of the stream functions increase with time and the vortices move in the direction opposite to the force of gravity and occupy the entire volume of the fluid. A two-vortex cell is formed; at the center of this cell the fluid rises and subsides at the thermally insulated edges. The increase in the magnitude of the stream functions and the extension of the vortices to the entire volume of the fluid is sufficiently rapid, only over a certain time interval, i.e., the induction period of the convection. The magnitude of the stream function and the nature of development of convection depend on the Rayleigh number.

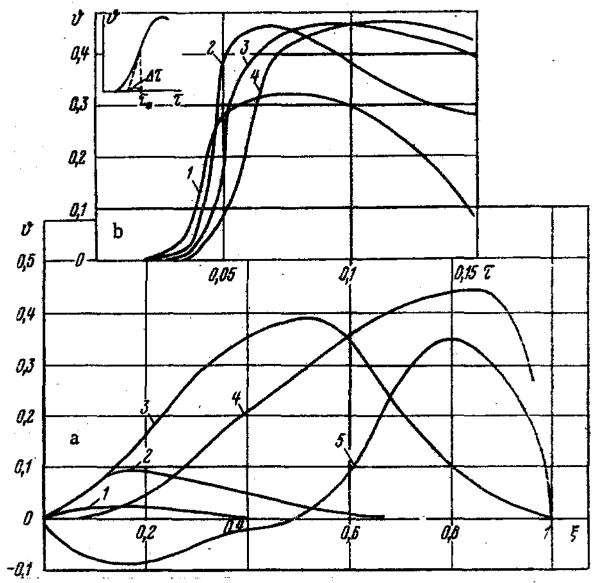

Fig. 3. Dependence of ϑ on ξ for $\eta = 0.5$ for different instants of time τ (a): 1) $\tau = 2\cdot10^{-2}$; 2) $3.5\cdot10^{-2}$; 3) $5\cdot10^{-2}$; 4) $1.25\cdot10^{-1}$; 5) $4\cdot10^{-1}$ and the dependence of ϑ on time for different ξ (b): 1) $\xi = 0.3$; 2) 0.5; 3) 0.7; 4) 0.8.

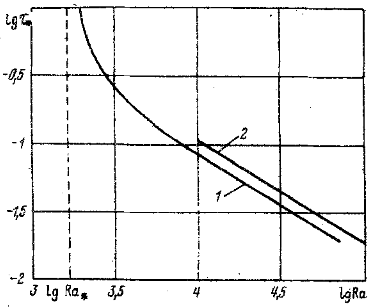

Fig. 4. Dependence of $\log \tau_*$ on $\log \mathrm{Ra}$: 1) computed on a computer; 2) curve computed from formula (1).

The dependence of θ on τ is shown in Fig. 2a for fixed η ($\eta = 0.5$) and different values of ξ. The dashed lines show the same dependence for Ra = 0. The temperature field for Ra = 0 is independent of η and corresponds to the field obtained from the solution of the problem of nonstationary thermal conductivity in a plane layer [3].

In the beginning the temperature in the fluid layer increases only due to thermal conductivity; later the convective motion develops at the lower boundary. The velocity of the fluid does not depend on ξ. Later on an inflection is observed on the heating curves, which corresponds to a strong contribution of the convective heat transport in the total transport of heat. This instant coincides with a sharp burst-type increase of the stream function $|\psi_m|$ in absolute magnitude (Fig. 2b). The most pronounced bending of the heating curves appears for sufficiently large ξ, i.e., for those fluid layers where the thermal conductivity makes almost no contribution to the heating. The temperature field has two symmetric regions in which the temperature variation in the vertical direction is small.

We shall characterize the effect of convection on the temperature field by the quantity

$$\vartheta = \theta(\eta, \xi, \tau, \mathrm{Ra}) - \hat{\theta}(\xi, \tau, 0),$$

where $\hat{\theta}$ is the solution of the problem for Ra = 0. The dependence of ϑ on ξ is shown in Fig. 3a for $\eta = 0.5$ and for different instants of time. Before a certain instant $\vartheta = 0$ for all ξ. As the convection develops ϑ becomes nonzero only for small values of ξ, i.e., the convection covers a narrow layer at the heated surface. With time the maximum of ϑ shifts toward larger ξ increasing in magnitude, i.e., the contribution of convection to heat transfer becomes significant in the vortex part of the fluid layer. The intensification of heat transfer from the lower heated layers of the fluid to the upper colder layers results in a decrease of ϑ in the region of small ξ (i.e., $\theta \to \hat{\theta}$).

Starting from a certain time instant ϑ changes with time quite sharply (for different ξ), which indicates a rapid replacement of the conduction regime of heat transfer by the convective regime. The latter permits to determine the period of induction of the convection development τ_*, starting from which convective heat transfer through the fluid layer plays an important role in the overall heat transfer. The method of determining τ_* is clear from Fig. 3a; τ_* determined in this way coincides with τ_* obtained from the dependence $|\psi_m|(\tau)$ (see Fig. 2b). The fact that τ_* has a weak dependence on ξ, permits to consider a common τ_* for the entire fluid layer. The quantity $\varepsilon = \Delta\tau/\tau_*$ (see Fig. 3a) reflects the nature of development of the convection. At values of Ra slightly different from Ra_* ε is large (of the order of 1) and for $\mathrm{Ra} = \mathrm{Ra}_*$ the period of induction of the convection is uncertain (i.e., in this case τ_* is equal to the time of heating of the entire fluid layer).

An investigation of the problem in conditions of Newtonian heat transfer at the vortex surface, i.e., for boundary conditions $\xi = 0$, $\theta = 0$, and $\xi = 1$, $-\partial\theta/\partial\xi = \mathrm{Bi}\,\theta$, showed that τ_* is practically independent of the Biot number (Bi). Physically this is quite clear, since the value of Bi should affect not only the development of convection but also the establishment of the stationary convective regime (for $\mathrm{Ra} > \mathrm{Ra}_*$).

The dependence of $\log \tau_*$ on $\log \mathrm{Ra}$ is shown in Fig. 4. It is evident from this figure that the results of the numerical computation are in good agreement with the experimental results. The computed curve 1 can be described by the following formula for Ra > 2000:

$$\tau_* = \frac{A\,\mathrm{Ra}^{1/3}}{\mathrm{Ra} - B}, \qquad (4)$$

where A = 36, B = 1500. For Ra ≫ B we obtain the dependence $\tau_* \sim \mathrm{Ra}^{-2/3}$. This practically corresponds to the values $\mathrm{Ra} > 10^4$. For Ra → B (B close to Ra_* in magnitude) $\tau_* \to \infty$.

NOTATION

- h is the height of the fluid layer;
- η, ξ are the dimensionless coordinates;
- τ is the dimensionless time;
- τ_* is the period of induction of convection development;

T	is the temperature;
θ	is the dimensionless temperature;
$\vec{v}$	is the velocity vector;
ρ	is the density;
ν	is the coefficient of kinematic viscosity;
a	is the coefficient of thermal conductivity;
$\vec{i}$	is the unit vector directed against the force of gravity;
$Pr = \nu/a$	is the Prandtl number;
Ra	is the Rayleigh number;
Ra_*	is the critical Rayleigh number;
Bi	is the Biot number.

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